

these techniques for bulb-temperature control and measurement were found to be responsible for the scatter shown in the lower curve of [1, fig. 3].

Therefore, a water-cooling system was devised which consisted of a small water jacket surrounding the lamp for about a 5-cm length in the vicinity of each cathode. The water was circulated through the jackets and an 8-l reservoir at a rate of 600 ml/min. The remainder of the bulb was kept warmer than the water-cooled portions by thermally insulating it; thus, the water temperature controlled the mercury vapor pressure within the lamp.<sup>1</sup>

#### MEASURING TECHNIQUES

The noise-temperature measuring technique was basically the same as in the original study. The lamp was inserted into three gold-plated copper tubes [1, fig. 2]; the outer two tubes were grounded and the noise was extracted from the center one via a double-stub tuner. The tuner was adjusted to give 50  $\Omega$  for each value of discharge current or bulb temperature by connecting it to an impedance bridge. After the impedance was adjusted, the noise output was measured by comparison with a corrected 5722 temperature-limited diode noise source. All measurements were made at 147 MHz.<sup>2</sup>

#### RESULTS

Fig. 1 shows the data obtained from one F8T5 lamp operated at 160-mA dc as the bulb temperature was varied from 70 to 0°C. One hundred and eight readings were taken in two separate runs several days apart. The standard deviation of the data is 0.02 dB (0.5 percent).

When compared with the data originally given for normal lamps [1, fig. 3], [2, fig. 1], the end points given here agree with the original values, but those for intermediate temperatures disagree, with the present values being higher. The slope for the portion from 40 to 70°C is  $-0.069$  dB/°C instead of  $-0.058$  dB/°C as originally reported.

In the original study, F8T5 and F13T5 lamps appeared to give the same noise temperature in spite of the different argon fill pressures so long as the lamps were operating with the normal amount of mercury. The present study shows a significant difference between lamp types which was previously hidden in the scatter. Curves similar to that given in Fig. 1 were obtained from two other lamps. One of the curves is about 0.1 dB higher at all temperatures; the other is about 0.05 dB higher. The variation from lamp to lamp agrees with known variations in the argon filling pressure.

During the tuning procedure with bulb temperatures between 28 and 70°C, the impedance bridge-detector output was about the same as with medium-pressure (20–50-mmHg) pure-argon discharges; i.e., very small impedance fluctuations existed. Below about 26°C, the detector "nulls" were similar to those from low-pressure (1–4-mmHg) pure-argon plasmas; i.e., there were large, random-impedance fluctuations having an average value equiv-

<sup>1</sup> Although the coldest spot on the bulb is supposed to control the mercury-vapor pressure, cataphoretic pumping, or some other effect, makes it necessary to control the bulb temperature at both ends of the lamp. Also, the noise temperature was somewhat erratic and low for up to 2 h after starting; this is believed to be caused by the liberation of adsorbed mercury from the phosphor in the hotter regions of the lamp.

<sup>2</sup> 147 MHz was used for the measurements because it is about the highest frequency at which temperature-limited diodes can be used without large corrections and the lowest frequency at which plasma sources can be used; i.e., traditional high-frequency noise sources and microwave noise sources can be compared in the upper VHF region.

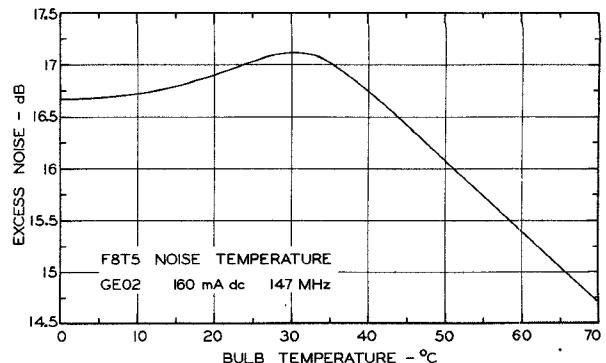


Fig. 1. Excess noise in decibels as a function of bulb temperature for a normal F8T5 lamp operating at 160-mA dc.  $EN_{dB} = 10 \log_{10}(T_N - 290)/290$ , where  $T_N$  is the noise (electron) temperature in kelvins.

alent to about a 4- $\Omega$  static-impedance error. The transition occurred over a small range in bulb temperature, typically from 26 to 28°C for the full change from a low- to a medium-pressure characteristic. This transition is slightly below the peak in the curve in Fig. 1, and optimum efficiency as a lamp occurs just beyond the peak.

#### CONCLUSION

The noise temperatures obtained from fluorescent lamps are quite reproducible when the bulb temperature, and thus the mercury-vapor pressure, is accurately controlled. Above 30°C, the characteristics are those of a medium-pressure discharge; the noise temperature is quite sensitive to bulb temperature and has a linear characteristic with a slope of  $-0.069$  dB/°C. Below a bulb temperature of 26°C the characteristics are those of a low-pressure argon discharge, and the noise temperature asymptotically approaches that of the rare-gas filling pressure.

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#### Comments on "Scattering of Surface Waves at a Dielectric Discontinuity on a Planar Waveguide"

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An analysis of the problem of scattering of surface waves at a dielectric discontinuity on a planar waveguide has recently appeared in this TRANSACTIONS.<sup>1</sup>

This writer has worked on some very similar problems and has reached somewhat different conclusions [1], [2]. The object

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<sup>1</sup> S. F. Mahmoud and J. C. Beal, *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-23, pp. 193–198, Feb. 1975.

of this letter is to compare between the approaches of footnote 1, [1] and [2], point out some differences, and make some suggestions which may lead towards obtaining better numerical results.

The authors of footnote 1 transform their equations (7), which are integral equations, into equations (9), which are then further transformed into an infinite system of algebraic equations (11). They achieve this transformation by expanding the two unknown functions  $\Gamma(\kappa)$  and  $T(\kappa)$  in a series of orthogonal functions [footnote 1, eq. (8)] appropriate for the domain  $0 \leq \kappa < \infty$  which occurs there.

The procedure, and the resulting system of equations (11) seem to be an obvious generalization of methods used for analyzing discontinuity problems in closed waveguides. There is, however, an essential difference between the cases of open and closed waveguides. In closed waveguides the entire field may be expressed as a linear combination of modes. Using orthogonality relations, an infinite set of linear algebraic equations for the mode-excitation coefficients is obtained. Physical considerations suffice to assure us that this set of equations has a unique solution. We further may accept the solution of a truncated set of equations as a good approximation, knowing that the mode expansion of the field in a waveguide is a rapidly convergent Fourier expansion. In open waveguides the field is expressed as a generalized linear combination of modes, i.e., as a finite sum of discrete modes, and an integral over the continuous spectrum of so-called pseudomodes. The mentioned functions  $\Gamma(\kappa)$  and  $T(\kappa)$  appear under the integral sign. Even though we have no doubt that the field representation, and resulting equations (7), are valid and bounded, there is no assurance that  $\Gamma$  and  $T$  are bounded for all  $0 \leq \kappa < \infty$ . As a matter of fact, there are strong indications to the contrary. Our investigations [1], [2] show that these functions satisfy singular integral equations, and have singular points for certain values of  $\kappa$ . Similar observations have been made by others in the investigation of different (but related) singular integral equations [3], [4].

Since the integrals over  $\Gamma$  and  $T$  exist, even though  $\Gamma$  and  $T$  are not necessarily bounded, the expansion of  $\Gamma$  and  $T$  in series of orthogonal functions [footnote 1, eq. (8)] is purely formal, and it may not be valid everywhere. As a result, it is not clear that the infinite system of algebraic equations (11) has a solution, or that the solution of a truncated finite subsystem thereof yields a good approximate solution (in some sense) to the original boundary-value problem.

Our approach in [1] and [2] is quite similar to footnote 1, except that we apply the inner product and use the orthogonality properties of the modes and pseudomodes directly to (7). Using [footnote 1, eq. (18)] it is readily seen from the terms in the denominator that the resulting integral equations for  $R$ ,  $T_i$ ,  $\Gamma(\kappa)$ , and  $T(\kappa)$  are singular, i.e., they have a kernel with a Cauchy-type singularity. We solve these equations asymptotically (for the case of a small discontinuity, i.e., for  $h/\lambda_0 \ll 1$ ), and obtain the form of the singularities in  $\Gamma$  and  $T$ . We suggest to try to solve the singular integral equations we get numerically, using a substitution which is similar to (8), namely,

$$\Gamma(\kappa) = g(\kappa) \sum_{j=0}^{\infty} \gamma_j f_j(\kappa)$$

$$T(\kappa) = g(\kappa) \sum_{j=0}^{\infty} t_j f_j(\kappa)$$

where  $\gamma_j$  and  $f_j$  are as in [footnote 1, eqs. (8) and (12)]. The function  $g(\kappa)$  should contain the singular parts of  $\Gamma$  and  $T$ .

Such an approach has been used in [4] for the numerical solution of a different (but related) singular integral equation. We have reason to believe that such an approach would be useful in our problem too. The form of  $g(\kappa)$  may be found by using the procedure of [1] and [2].

Reply<sup>2</sup> by Samir F. Mahmoud<sup>3</sup>

The problem treated in footnote 1 is that of scattering of a surface wave at an abrupt dielectric discontinuity on the guiding structure (Fig. 1). The solution in footnote 1 is based on the expansion of the electric and magnetic fields on both guides  $a$  and  $b$  into their normal modes. These are composed of a finite number of surface-wave modes plus the radiation field which is a continuous spectrum of pseudomodes whose normalized transverse wavenumber  $\kappa$  (in the  $y$  direction) takes all values between 0 and  $\infty$ . It is known that a single pseudomode does not satisfy the radiation condition since it behaves as the summation of two plane waves as  $y \rightarrow \infty$ . However, the integrated continuous spectrum of pseudomodes does satisfy this condition as it should [5].

The situation in Fig. 1 can be looked at as the excitation of guide  $b$  by an incident surface-wave mode from guide  $a$ . Since the incident fields satisfy the radiation condition, the transmitted fields on guide  $b$  must also satisfy this condition. This requires that no singularity should exist in the continuous spectrum since such a singularity, which corresponds to a single pseudomode, signifies that the total transmitted fields contain a plane wave at infinity and hence violate the radiation condition. Furthermore no finite number of such singularities can exist without violating this condition. However, should the incident fields on guide  $a$  include any singularity in their continuous spectrum, such as the case of an incident plane wave, we would have expected both reflected and transmitted plane waves to be excited at the discontinuity plane. In such case, the fields on both sides of the discontinuity would not satisfy the radiation condition.

A mathematical proof of the previous intuitive discussion can be given as follows. For convenience we shall rewrite [footnote 1, eq. (7)] which expresses the continuity of tangential fields at  $z = 0$  and  $-t \leq y \leq \infty$ .

$$(1 + R)e_1^a + \int_0^{\infty} \Gamma(\kappa)e^a(\kappa) dk = \sum_{i=1}^{M_s} T_s e_i^b + \int_0^{\infty} T(\kappa)e^b(\kappa) dk$$

$$(1 - R)h_1^a - \int_0^{\infty} \Gamma(\kappa)h^a(\kappa) dk = \sum_{i=1}^{M_s} T_s h_i^b + \int_0^{\infty} T(\kappa)h^b(\kappa) dk$$

where the modal fields  $e$  and  $h$  are functions of the transverse coordinate  $y$  as well as the wavenumber  $\kappa$ . Now, we multiply the first equation by  $h^b(\kappa')$ , the second by  $e^b(\kappa')$ , integrate both equations  $w \cdot r \cdot t \cdot y$  over the range  $-t \leq y < \infty$  and use the modal orthogonality relations in footnote 1 to obtain

$$(1 + R)\langle e_1^a, h^b(\kappa') \rangle + \int_0^{\infty} \Gamma(\kappa)\langle e^a(\kappa), h^b(\kappa') \rangle dk = T(\kappa')N^b(\kappa') \quad (1)$$

<sup>2</sup> Manuscript received January 26, 1976.

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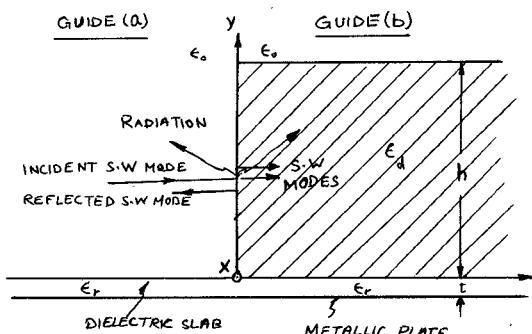


Fig. 1. Geometry of the problem treated in footnote 1.

$$(1 - R) \langle e^b(\kappa'), h_1^a \rangle - \int_0^\infty \Gamma(\kappa) \langle e^b(\kappa'), h^a(\kappa) \rangle d\kappa = T(\kappa') N^b(\kappa') \quad (2)$$

where  $\langle f, g \rangle$  is an inner product defined in footnote 1 and involves an integration  $w \cdot r \cdot t \cdot y \cdot N_b(\kappa')$  is given by

$$2\pi\beta(\kappa') V^b(\kappa') W^b(\kappa')$$

where the quantities  $\beta$ ,  $V$ , and  $W$  are defined by [footnote 1, eqs. (1)-(3)]. The inner product  $\langle e^a(\kappa), h^b(\kappa') \rangle$  is given explicitly by [footnote 1, eq. (18)]. Here we need only to write it in the form

$$\begin{aligned} \langle e^a(\kappa), h^b(\kappa') \rangle &= C_1(\kappa') \delta(\kappa - \kappa') \\ &+ (1 - \kappa^2)^{1/2} C_2(\kappa, \kappa') / (\kappa - \kappa') + f_1(\kappa, \kappa'). \end{aligned} \quad (3)$$

Similarly,

$$\begin{aligned} \langle e^b(\kappa'), h^a(\kappa) \rangle &= C_1(\kappa') \delta(\kappa - \kappa') \\ &+ (1 - \kappa'^2)^{1/2} C_2(\kappa, \kappa') / (\kappa - \kappa') + f_2(\kappa, \kappa') \end{aligned} \quad (4)$$

where  $\delta(\cdot)$  is the Kronecker  $\delta$ -function,  $C_1(\kappa')$  and  $C_2(\kappa, \kappa')$  are given by

$$\begin{aligned} C_1(\kappa') &= 2\pi(1 - \kappa'^2)^{1/2} [V^a(\kappa') W^b(\kappa') + W^a(\kappa') V^b(\kappa')] \\ C_2(\kappa, \kappa') &= -i[V^a(\kappa) W^b(\kappa') - W^a(\kappa) V^b(\kappa')] \end{aligned}$$

and  $f_{1,2}(\kappa, \kappa')$  are finite functions which can be obtained from [footnote 1, eq. (18)]. Now we substitute from (3) and (4) in (1) and (2) and integrate the singular terms separately to obtain

$$\begin{aligned} (1 + R) \langle e_1^a, h^b(\kappa') \rangle + C(\kappa') \Gamma(\kappa') + \int_0^\infty \Gamma(\kappa) F_1(\kappa, \kappa') d\kappa &= T(\kappa') N^b(\kappa') \end{aligned} \quad (5)$$

$$\begin{aligned} (1 - R) \langle e^b(\kappa'), h_1^a \rangle - C(\kappa') \Gamma(\kappa') - \int_0^\infty \Gamma(\kappa) F_2(\kappa, \kappa') d\kappa &= T(\kappa') N^b(\kappa') \end{aligned} \quad (6)$$

where

$$C(\kappa') = C_1(\kappa') + 2\pi i R_s(\kappa')$$

and  $R_s(\kappa') \Gamma(\kappa')$  is the residue resulting from the integration of the term including  $1/(\kappa - \kappa')$  around the singularity  $\kappa = \kappa'$  after the appropriate modification of the contour of integration. The functions  $F_{1,2}(\kappa, \kappa')$  represent the terms in (3) and (4) other than the singularities; i.e., they are finite for all values of  $\kappa$  and  $\kappa'$ . Furthermore,  $F_{1,2}(\kappa, \kappa')$  behave as  $\kappa^{-1}$  as  $\kappa \rightarrow \infty$  with  $\kappa'$  finite as can be proved from [footnote 1, eq. (18)]. From the

radiation condition it can be easily shown that  $\Gamma(\kappa)$  tapers off as  $\kappa^{-1/2}$  or faster as  $\kappa \rightarrow \infty$ . Hence, the integrals in (5) and (6) are finite. The first terms on the LHS of these equations are also finite by virtue of the exponential decay of the surface-wave fields. Hence (5) and (6) can be cast in the forms

$$\begin{aligned} N^b(\kappa') T(\kappa') &= C(\kappa') \Gamma(\kappa') + \text{a finite quantity } X_1(\kappa') \\ N^b(\kappa') T(\kappa') &= -C(\kappa') \Gamma(\kappa') + \text{a finite quantity } X_2(\kappa'). \end{aligned}$$

By adding and subtracting these two equations, we conclude that both  $T(\kappa')$  and  $\Gamma(\kappa')$  are finite<sup>4</sup> quantities for all finite values of  $\kappa'$ . We should note, however, that at  $\kappa' = 1$ , (6) becomes identically zero and no definite conclusion can be made about the finiteness of  $\Gamma(1)$  and  $T(1)$ . We may then resort to the radiation condition which necessitates the finiteness of these quantities.

The previous discussion, we believe, constitutes a proof of the absence of any singularity in the continuous spectra of the fields on both sides of the discontinuity, and hence the eligibility of expanding  $\Gamma(\kappa)$  and  $T(\kappa)$  in terms of Laguerre polynomials as used in footnote 1.

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<sup>4</sup> In [2] Rulf and Kedem adopted a fixed-order approximation for  $T(\kappa)$  and  $\Gamma(\kappa)$  in terms of a small parameter of discontinuity. We believe that the resulting singularities in their solution should be removed by taking a better approximation.

## Comments on "Asymmetric Coupled Transmission Lines in an Inhomogeneous Medium"

EUGENIO COSTAMAGNA AND UGO MALTESE

In the above paper,<sup>1</sup> the terminal characteristic parameters for a uniform coupled-line four-port for the general case of an asymmetric, inhomogeneous system are derived, and some of the equivalent circuits are presented.

We have read with interest the paper, in particular the discussions on the behavior of the modes of the structure.

An alternative method for expressing the propagation constants and the terminal parameters is the use of the capacitances in air and in inhomogeneous dielectric for odd and even excitation. This is very useful in the analysis and optimization of distributed networks. Formulas and procedures have been given in [1] with equivalent circuits for comb line and interdigital sections.

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<sup>1</sup> V. K. Tripathi, *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-23, pp. 734-739, Sept. 1975.